

# Muffler Optimization

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14-05-2015



- ▶ Maximization of muffler performance is important, but there is always space volume constraints
- ▶ Shape optimization of multi-segments Muffler coupled with the GA searching technique

## Outline

- ▶ Problem Statement
- ▶ Derivation of Four Pole Matrices and an expression for STL
- ▶ Introduction to GA and it's Implementation
- ▶ A numerical case of noise elimination on pure tone
- ▶ Results and Discussion



# Problem Statement

- ▶ The available space for muffler is 0.3 m (Length) ,0.3 m (Width) and 1 m (Height)
- ▶ To reduce the pure tone noise effectively, four kinds of multi-segments mufflers (2–5 segments) are proposed

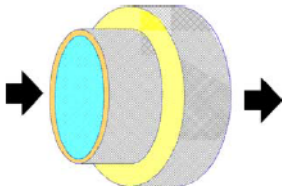


Figure 1: Three-dimensional cross-section for two-segments muffler



# Multi-segment Mufflers

- ▶ Four kinds of multi-segments mufflers are graphically depicted

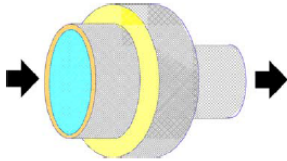


Figure 2: Three-dimensional cross-section for three-segments muffler

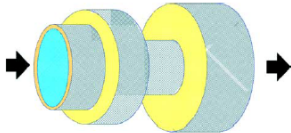


Figure 3: Three-dimensional cross-section for Four-segments muffler



# Space Constraints

- ▶ The related boundary constraints for the mufflers are specified.

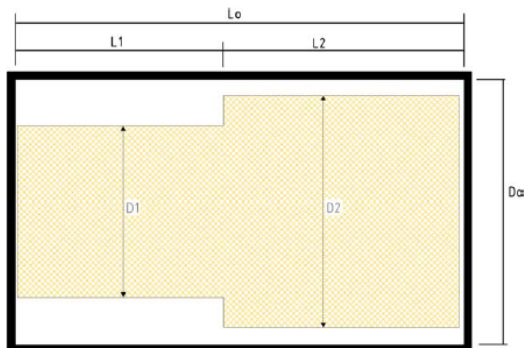


Figure 4: Space constraints for two-segments muffler [ $L_0 = 0.3m$ ,  $D_0 = 0.3m$ ]



# Derivation of Four Pole Matrices and an expression for STL

- ▶ For the ease of theoretical derivation on muffler, two kinds of muffler elements, are identified,
- ▶ On the basis of plane wave theorem, a transfer matrix between inlet and outlet can then be deduced in each muffler element

$$\begin{aligned} \begin{pmatrix} p_1 \\ \rho_0 c_0 u_1 \end{pmatrix} &= \exp \left\{ -j \frac{M_1 k L}{1 - M_1^2} \right\} \begin{bmatrix} \cos \left( \frac{k L_1}{1 - M_1^2} \right) & j \sin \left( \frac{k L_1}{1 - M_1^2} \right) \\ j \sin \left( \frac{k L_1}{1 - M_1^2} \right) & \cos \left( \frac{k L_1}{1 - M_1^2} \right) \end{bmatrix} \begin{pmatrix} p_2 \\ \rho_0 c_0 u_2 \end{pmatrix} \\ &= \exp \left\{ -j \frac{M_1 k L}{1 - M_1^2} \right\} \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix} \begin{pmatrix} p_2 \\ \rho_0 c_0 u_2 \end{pmatrix}, \end{aligned}$$

where

$$\begin{aligned} b_{11}^* &= \cos \left( \frac{k L_1}{1 - M_1^2} \right); & b_{12}^* &= j \sin \left( \frac{k L_1}{1 - M_1^2} \right); \\ b_{21}^* &= j \sin \left( \frac{k L_1}{1 - M_1^2} \right); & b_{22}^* &= \cos \left( \frac{k L_1}{1 - M_1^2} \right). \end{aligned}$$

Figure 5: Four poles matrix between point 1 and point 2 with mean flow



## Derivation of Four Pole Matrices...2

Four poles matrix between point 2 and point 3 with mean flow is:

$$\begin{pmatrix} p_2 \\ \rho_0 c_0 u_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{S_2}{S_1} \end{bmatrix} \begin{pmatrix} p_3 \\ \rho_0 c_0 u_3 \end{pmatrix}.$$

Figure 6: Four poles matrix between point 2 and point 3 with mean flow

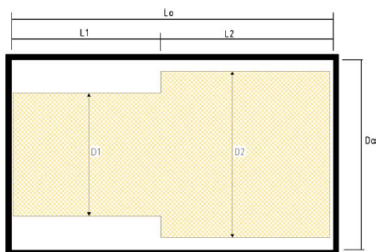


Figure 7: Space constraints for two-segments muffler



## Derivation of Expression for STL....3

After multiplying all the above matrices, we will obtain the final transfer matrix

$$\begin{bmatrix} p_1 \\ \rho_0 c_0 u_1 \end{bmatrix} = \begin{bmatrix} T_{11}^* & T_{12}^* \\ T_{21}^* & T_{22}^* \end{bmatrix} \times \begin{bmatrix} p_4 \\ \rho_0 c_0 u_4 \end{bmatrix}$$

The sound transmission loss (STL) of muffler is defined as

$$\text{STL} = 20 \log \left( \frac{|T_{11}^* + T_{12}^* + T_{21}^* + T_{22}^*|}{2} \right) + 10 \log \left( \frac{S_1}{S_{10}} \right).$$

Figure 8: Final expression for STL





# Genetic Algorithm

- ▶ Search algorithms based on the mechanics of natural selection and natural genetics
- ▶ Based on “survival of fittest” concept
- ▶ Simulates the process of evolution
- ▶ KEY IDEA: “Evolution is an optimizing process”

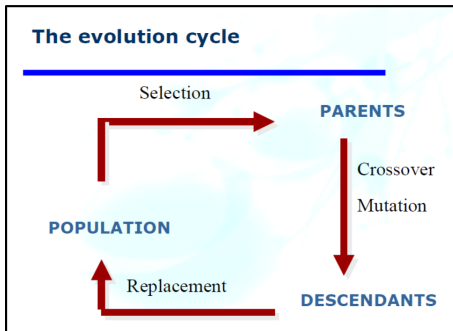


Figure 9: The Evolution cycle



# Genetic Algorithm : Initialization

- ▶ Population, whose individuals represent solution to problems
- ▶  $(d_1, d_2) = (5.4064, 3.8005)$  is a member in our population!
- ▶ A member/Design vector  $(d_1, d_2) = (5.4064, 3.8005)$  may be represented using binary numbers like this

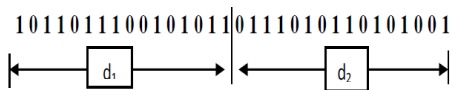
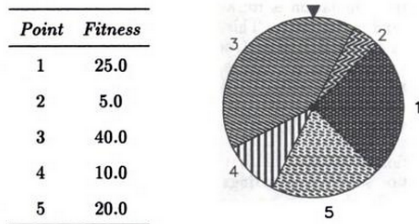


Figure 10: Design vector coded to string structure



# Genetic Algorithm : Ranking the Genomes

- ▶ Each individual/ String is evaluated to find the fitness value
- ▶ Roulette Wheel Selection is implemented



**Figure 11:** A roulette wheel marked for five individuals according to their fitness [ Figure Courtesy: Optimization for Engineering Design: Algorithms and Examples, Kalyanmoy Deb ]



# Genetic Algorithm : Reproduction Operators

## Single Point Crossover

- ▶ Each chromosome of parent is divided into two parts and then joined stochastically

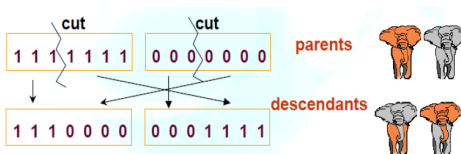


Figure 12: Single point Crossover

## Mutation

- ▶ To make sure that sufficient variety of strings are there to assure that GA will go through the entire problem space
- ▶ Prevents premature convergence



## Elitism

- ▶ The elitism scheme to keep best gene in the parent generations
- ▶ To prevent the best gene from the disappearing and improve the accuracy of optimization during reproduction



# A numerical case of noise elimination

- ▶ With the spectrum analysis in sound, it is found that the sound energy at 500 Hz is highly remarkable.
- ▶ The minimal diameters at each segment are specified to be no less than 0.0762 m
- ▶ The design volume flow rate is confined to 0.8 CMS.
- ▶ For optimization of a Two segments muffler, 3 parameters were selected
  - ▶ Diameter,  $D_1$
  - ▶ Diameter,  $D_2$
  - ▶ Length,  $L_1$



# Results and Discussion

- ▶ The maximal value of STL is 38.5 dB

Table 1  
Results comparison for two-segments muffler

	Common parameters			Control parameters			Results				Elapsed time, $t$ (min)
	PopuSize	Gen_no	Bit_no	pc	pm	Elt_no	$D_1$ (m)	$D_2$ (m)	$L_1$ (m)	STL	
Case 1	60	500	40	0.8	0.05	1	0.3000	0.0762	0.1424	38.5	0.70
Case 2	60	500	40	0.8	0.05	0	0.2982	0.0764	0.8395	38.4	0.69
Case 3	60	500	40	0.8	0	1	0.2478	0.0801	0.1500	35.5	0.70
Case 4	60	500	40	0	0.05	1	0.2992	0.0790	0.1566	37.5	0.65

Figure 13: Tabulation of finally obtained results

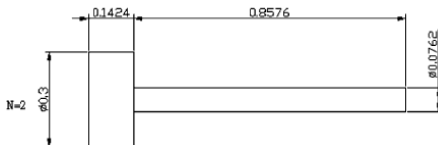


Figure 14: Optimal shape in a two segment muffler



## Results and Discussion...2

- ▶ The performance curves for different GA control parameter are plotted.

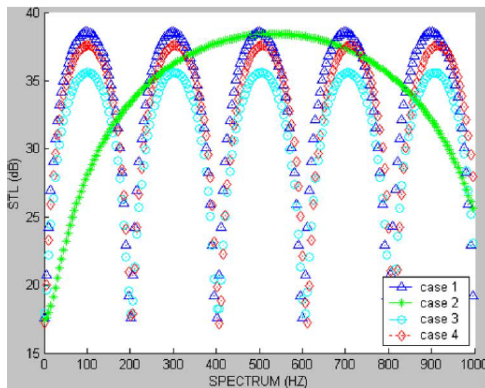


Figure 15: STL of two-segments muffler at four sets of GA parameters.





- ▶ Because of no first derivative and starting design data of objective function as required in traditional gradient method, GA becomes easier.
- ▶ The case study reveals that by increasing the segments in muffler, the performance in STL can be improved efficiently.
- ▶ Results are sensitive to the GA control parameters like, probability of crossover  $p_c$  and probability of mutation  $p_m$



# Thanks

